

Unified Description and Canonical Reduction to Dirac's Observables of the Four Interactions

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I. SYSTEMS WITH CONSTRAINTS.

The standard $SU(3) \times SU(2) \times U(1)$ model of elementary particles, all its extensions with or without supersymmetry, all variants of string theory, all formulations of general relativity are described by singular Lagrangians. Therefore, their Hamiltonian formulation needs Dirac's theory of 1st and 2nd class constraints [1] determining the submanifold of phase space relevant for dynamics: this means that the basic mathematical structure behind our description of the four interactions is presymplectic geometry (namely the theory of submanifolds of phase space with a closed degenerate 2-form; strictly speaking only 1st class constraints are associated with presymplectic manifolds: the 2nd class ones complicate the structure). For a system with 1st and 2nd class constraints the physical description becomes clear in coordinates adapted to the presymplectic submanifold. Locally in phase space, an adapted Darboux chart can always be found by means of Shanmugadhasan's canonical transformations [2] [strictly speaking their existence is proved only for finite-dimensional systems, but they underlie the existence of the Faddeev-Popov measure for the path integral]. The new canonical basis has: i) as many new momenta as 1st class constraints (Abelianization of 1st class constraints); ii) their conjugate canonical variables (Abelianized gauge variables);

iii) as many pairs of canonical variables as pairs of 2nd class constraints (standard form, adapted to the chosen Abelianization, of the irrelevant variables); iv) pairs of canonically conjugate Dirac's observables (canonical basis of physical variables adapted to the chosen Abelianization; they give a trivialization of the BRST construction of observables). Putting equal to zero the Abelianized gauge variables one defines a local gauge of the model. If a system with constraints admits one (or more) global Shanmugadhasan canonical transformations, one obtains one (or more) privileged global gauges in which the physical Dirac observables are globally defined and globally separated from the gauge and the irrelevant degrees of freedom [for systems with a compact configuration space this is impossible]. These privileged gauges (when they exist) can be called generalized Coulomb gauges.

To find them the main problem is to discover how the original canonical variables depend upon the gauge variables. This can be achieved by solving (if possible) the so-called multitemporal equations [see Refs. [3]a), b)]: by considering each of the original 1st class constraints as the Hamiltonian for the evolution in a suitable parameter (called a generalized time), these equations are the associated Hamilton equations. If one succeeds in solving these equations (which are formally integrable due the 1st class nature of the constraints), one finds the the suitable parameters are just the Abelianized gauge variables and, then, one can construct the conjugate Abelianized 1st class constraints and the standard form of the 2nd class ones. Every set of suitable generalized times gives rise to a different generalized Coulomb gauge. Let us remark that in certain cases it is possible to find some special global Shanmugadhasan canonical transformations such that the effective new Hamiltonian of the system is automatically the sum of the physical Hamiltonian (depending only on Dirac's observables) and a gauge Hamiltonian (depending only on the gauge variables and the 1st class constraints): in these cases there is a decoupling of the gauge degrees of freedom without the need to add gauge-fixings (i.e. without putting to zero the Abelianized gauge variables).

Therefore, given a system with constraints without a compact configuration space, one has to investigate whether there is any obstruction to the existence of global Shanmugadhasan canonical transformations, namely whether one can do a global canonical reduction.

Possible obstructions can arise when the configuration (and then also the phase) space and/or related mathematical structures like fiber bundles are not topologically trivial: usually they may be present when certain cohomological groups of the classical manifolds are not trivial (these groups are at the basis of the possible existence of anomalies in the quantization of the system). A related problem in gauge theories is the possible existence of Gribov-type ambiguities (nontrivial stability groups of gauge transformations for gauge potentials and/or field strengths): since they imply the nonexistence of global gauges, they destroy the possibility of a global decoupling of physical and gauge degrees of freedom. These ambiguities imply that both the configuration space and the phase space constraint manifold are in general stratified manifolds (i.e. disjoint union of manifolds) possibly with singularities. Moreover, for constrained systems defined in Minkowski spacetime the constraint manifold is always a stratified manifold, because one always assumes that the kinematical Poincaré group is globally implemented for isolated systems. This implies that the ten Poincaré generators must be finite (this is a first restriction on the boundary conditions of the fields present in the isolated system to allow use of group theory) and that the constraint manifold is the disjoint union of manifolds, each one of which contains all the system configurations belonging to the same type of Poincaré orbit (spacelike orbits should be absent at the classical level not to have causality problems). The main stratum, dense in the constraint manifold, will be the one associated with nonzero spin (i.e. with nonzero Pauli-Lubanski Casimir) timelike orbits. The existence of this Poincaré stratification raises the general question whether there could exist generalized Coulomb gauges with some kind of manifest covariance under Lorentz transformations and with some kind of universal breaking of manifest Lorentz covariance (unavoidable when one eliminates all the gauge degrees of freedom).

Moreover, one would like to have of the results obtained in Minkowski spacetime M^4 described in a form which can be extended to incorporate general relativity.

Given this general setting for constrained systems, a research program started trying to get a description only in terms of Dirac's observables and with an explicit control on covariance of (to start with) the $SU(3) \times SU(2) \times U(1)$ standard model of elementary

particles coupled to tetrad gravity (more natural than metric gravity for the coupling to fermion fields). See Refs. [3] for the genesis and the developments of this program, which is well defined only for isolated systems [to recover theories with external fields one should consider special limits of some parameter of some subsystem].

In the next Sections a review of the results will be given.

II. NONCOVARIANT GENERALIZED COULOMB GAUGES.

Dirac [4] found the Coulomb gauge physical Hamiltonian of the isolated system formed by a fermion field plus the electromagnetic field [see the second paper in Ref. [5] for the case of Grassmann-valued fermion fields], which contains the coupling of physical fermions with the radiation field and the nonlocal Coulomb self-energy of the fermion field: $\int d^3x d^3y (\check{\psi}^\dagger \check{\psi})(\vec{x}, x^o) \frac{1}{4\pi|\vec{x}-\vec{y}|} (\check{\psi}^\dagger \check{\psi})(\vec{y}, x^o)$. The Dirac observables are the transverse vector potential $\vec{A}_\perp(\vec{x}, x^o)$, the transverse electric field $\vec{E}_\perp(\vec{x}, x^o)$ and physical fermion fields dressed with a Coulomb cloud, $\check{\psi}(\vec{x}, x^o) = e^{i\eta_{em}(\vec{x}, x^o)} \psi(\vec{x}, x^o)$, $\eta_{em} = -\frac{1}{\Delta} \vec{\partial} \cdot \vec{A}$.

Extending this approach, the generalized Coulomb gauge of the following isolated systems has been found [see Ref. [3] c) for other systems like the Nambu string, relativistic two-body systems with action-at-a-distance interactions and for nonrelativistic Newton mechanics reformulated with 1st class constraints]:

a) Yang-Mills theory with Grassmann-valued fermion fields [5] in the case of a trivial principal bundle over a fixed- x^o R^3 slice of Minkowski spacetime with suitable Hamiltonian-oriented boundary conditions; this excludes monopole solutions and, since R^3 is not compactified, one has only winding number and no instanton number. After a discussion of the Hamiltonian formulation of Yang-Mills theory, of its group of gauge transformations and of the Gribov ambiguity, the theory has been studied in suitable weighted Sobolev spaces where the Gribov ambiguity is absent. The global Dirac observables are the transverse quantities $\vec{A}_{a\perp}(\vec{x}, x^o)$, $\vec{E}_{a\perp}(\vec{x}, x^o)$ and fermion fields dressed with Yang-Mills (gluonic) clouds. The nonlocal and nonpolynomial (due to classical Wilson lines along flat geodesics)

physical Hamiltonian has been obtained: it is nonlocal but without any kind of singularities, it has the correct Abelian limit if the structure constants are turned off, and it contains the explicit realization of the abstract Mitter-Viallet metric.

b) The Abelian and non-Abelian $SU(2)$ Higgs models with fermion fields [6,7], where the symplectic decoupling is a refinement of the concept of unitary gauge. There is an ambiguity in the solutions of the Gauss law constraints, which reflects the existence of disjoint sectors of solutions of the Euler-Lagrange equations of Higgs models. The physical Hamiltonian and Lagrangian of the Higgs phase have been found; the self-energy turns out to be local and contains a local four-fermion interaction.

c) The standard $SU(3) \times SU(2) \times U(1)$ model of elementary particles [8] with Grassmann-valued fermion fields. The final reduced Hamiltonian contains nonlocal self-energies for the electromagnetic and color interactions, but “local ones” for the weak interactions implying the nonperturbative emergence of 4-fermions interactions. To obtain a nonlocal self-energy with a Yukawa kernel for the massive Z and W^\pm bosons one has to reformulate the model on spacelike hypersurfaces and make a modification of the Lagrangian.

III. WIGNER-COVARIANT REST-FRAME INSTANT FORM.

The next problem is how to covariantize these results. Again the starting point was given by Dirac [1] with his reformulation of classical field theory on spacelike hypersurfaces foliating Minkowski spacetime. In this way one gets parametrized field theory with a covariant 3+1 splitting of flat spacetime and already in a form suited to the coupling to general relativity in its ADM canonical formulation (see also Ref. [9] , where a theoretical study of this problem is done in curved spacetimes) The price is that one has to add as new configuration variables the points $z^\mu(\tau, \vec{\sigma})$ of the spacelike hypersurface Σ_τ [the only ones carrying Lorentz indices; the scalar parameter τ labels the leaves of the foliation and $\vec{\sigma}$ are curvilinear coordinates on Σ_τ] and then to define the fields on Σ_τ so that they know the hypersurface Σ_τ of τ -simultaneity [for a Klein-Gordon field $\phi(x)$ this new field is $\tilde{\phi}(\tau, \vec{\sigma}) = \phi(z(\tau, \vec{\sigma}))$]. Then,

besides a Lorentz-scalar form of the constraints of the given system, from the Lagrangian rewritten on the hypersurface [function of z^μ through the induced metric $g_{\check{A}\check{B}} = z_{\check{A}}^\mu \eta_{\mu\nu} z_{\check{B}}^\nu$, $z_{\check{A}}^\mu = \partial z^\mu / \partial \sigma^{\check{A}}$, $\sigma^{\check{A}} = (\tau, \sigma^{\check{r}})$] one gets 4 further first class constraints $\mathcal{H}_\mu(\tau, \vec{\sigma}) \approx 0$ implying the independence of the description from the choice of the spacelike hypersurfaces. Being in special relativity, it is convenient to restrict ourselves to arbitrary spacelike hyperplanes $z^\mu(\tau, \vec{\sigma}) = x_s^\mu(\tau) + b_{\check{r}}^\mu(\tau) \sigma^{\check{r}}$. Since they are described by only 10 variables [an origin $x_s^\mu(\tau)$ and 3 orthogonal spacelike unit vectors generating the fixed constant timelike unit normal to the hyperplane], we remain only with 10 first class constraints determining the 10 variables conjugate to the hyperplane [they are a 4-momentum p_s^μ and the 6 independent degrees of freedom hidden in a spin tensor $S_s^{\mu\nu}$] in terms of the variables of the system.

If we now restrict ourselves to timelike ($p_s^2 > 0$) 4-momenta, we can restrict the description to the so-called Wigner hyperplanes orthogonal to p_s^μ itself. To get this result, we must boost at rest all the variables with Lorentz indices by using the standard Wigner boost $L^\mu{}_\nu(p_s, \vec{p}_s)$ for timelike Poincaré orbits, and then add the gauge-fixings $b_{\check{r}}^\mu(\tau) - L^\mu{}_{\check{r}}(p_s, \vec{p}_s) \approx 0$. Since these gauge-fixings depend on p_s^μ , the final canonical variables, apart p_s^μ itself, are of 3 types: i) there is a non-covariant center-of-mass variable $\tilde{x}^\mu(\tau)$ [the classical basis of the Newton-Wigner position operator]; ii) all the 3-vector variables become Wigner spin 1 3-vectors [boosts in M^4 induce Wigner rotations on them]; iii) all the other variables are Lorentz scalars. Only the 4 1st class constraints determining p_s^μ are left. One obtains in this way a new kind of instant form of the dynamics (see Ref. [10]), the Wigner-covariant 1-time rest-frame instant form [11] with a universal breaking of Lorentz covariance. It is the special relativistic generalization of the nonrelativistic separation of the center of mass from the relative motion [$H = \frac{\vec{P}^2}{2M} + H_{rel}$]. The role of the center of mass is taken by the Wigner hyperplane, identified by the point $\tilde{x}^\mu(\tau)$ and by its normal p_s^μ . The 4 first class constraints can be put in the following form: i) the vanishing of the total (Wigner spin 1) 3-momentum of the system $\vec{p}[\text{system}] \approx 0$, saying that the Wigner hyperplane $\Sigma_W(\tau)$ is the intrinsic rest frame [instead, \vec{p}_s is left arbitrary, since it reflects the orientation of the Wigner hyperplane with respect to arbitrary reference frames in M^4];

ii) $\pm\sqrt{p_s^2} - M[\text{system}] \approx 0$, saying that the invariant mass M of the system replaces the nonrelativistic Hamiltonian H_{rel} for the relative degrees of freedom, after the addition of the gauge-fixing $T_s - \tau \approx 0$ [identifying the time parameter τ with the Lorentz scalar time of the center of mass in the rest frame; M generates the evolution in this time]. When one is able, as in the case of N free particles [11], to find the (Wigner spin 1) 3-vector $\vec{\eta}(\tau)$ conjugate to $\vec{p}[\text{system}](\approx 0)$, the gauge-fixing $\vec{\eta} \approx 0$ eliminates the gauge variables describing the 3-dimensional intrinsic center of mass inside the Wigner hyperplane [$\vec{\eta} \approx 0$ forces it to coincide with $x_s^\mu(\tau) = z^\mu(\tau, \vec{\sigma} = \vec{\eta} = 0)$ and breaks the translation invariance $\vec{\sigma} \mapsto \vec{\sigma} + \vec{a}$], so that we remain only with Newtonian-like degrees of freedom with rotational covariance:

i) a 3-coordinate (not Lorentz covariant) $\vec{z}_s = \sqrt{p_s^2}(\vec{x}_s - \frac{\vec{p}_s}{p_s^0} \hat{x}^0)$ and its conjugate momentum $\vec{k}_s = \vec{p}_s / \sqrt{p_s^2}$ for the absolute center of mass in M^4 ; ii) a set of relative conjugate pairs of variables with Wigner covariance inside the Wigner hyperplane .

The systems till now analyzed to get their rest-frame generalized Coulomb gauges are:

a) The system of N scalar particles with Grassmann electric charges plus the electromagnetic field [11]. The starting configuration variables are a 3-vector $\vec{\eta}_i(\tau)$ for each particle [$x_i^\mu(\tau) = z^\mu(\tau, \vec{\eta}_i(\tau))$] and the electromagnetic gauge potentials $A_{\hat{A}}(\tau, \vec{\sigma}) = \frac{\partial z^\mu(\tau, \vec{\sigma})}{\partial \sigma^{\hat{A}}} A_\mu(z(\tau, \vec{\sigma}))$, which know implicitly the embedding of Σ_τ into M^4 . One has to choose the sign of the energy of each particle, because there are not mass-shell constraints (like $p_i^2 - m_i^2 \approx 0$) among the constraints of this formulation, due to the fact that one has only 3 degrees of freedom for particle, determining the intersection of a timelike trajectory and of the spacelike hypersurface Σ_τ . The final Dirac's observables are: i) the transverse radiation field variables; ii) the particle canonical variables $\vec{\eta}_i(\tau)$, $\vec{\kappa}_i(\tau)$, dressed with a Coulomb cloud. The physical Hamiltonian contains the Coulomb potentials extracted from field theory and there is a regularization of the Coulomb self-energies due to the Grassmann character of the electric charges Q_i [$Q_i^2 = 0$]. In Ref. [12] there is the study of the Lienard-Wiechert potentials and of Abraham-Lorentz-Dirac equations in this rest-frame Coulomb gauge and also scalar electrodynamics is reformulated in it. Also the rest-frame 1-time relativistic statistical

mechanics is developed [11].

b) The system of N scalar particles with Grassmann-valued color charges plus the color $SU(3)$ Yang-Mills field [13]: it gives the pseudoclassical description of the relativistic scalar-quark model, deduced from the classical QCD Lagrangian and with the color field present. The physical invariant mass of the system is given in terms of the Dirac observables. From the reduced Hamilton equations the second order equations of motion both for the reduced transverse color field and the particles are extracted. Then, one studies the $N=2$ (meson) case. A special form of the requirement of having only color singlets, suited for a field-independent quark model, produces a “pseudoclassical asymptotic freedom” and a regularization of the quark self-energy. With these results one can covariantize the bosonic part of the standard model given in Ref. [8].

c) It is in an advanced stage the description of Dirac and chiral fields and of spinning particles on spacelike hypersurfaces [14]. After its completion, the rest-frame form of the full standard $SU(3) \times SU(2) \times U(1)$ model can be achieved.

Finally, to eliminate the three 1st class constraints $\vec{p}[\text{system}] \approx 0$ by finding their natural gauge-fixings, when fields are present, one needs to find a rest-frame canonical basis of center-of-mass and relative variables for fields (in analogy to particles). Such a basis has already been found for a real Klein-Gordon field [15]. This kind of basis will allow, after quantization, to find the asymptotic states of the covariant Tomonaga-Schwinger formulation of quantum field theory on spacelike hypersurfaces: these states are needed for the theory of quantum bound states [since Fock states do not constitute a Cauchy problem for the field equations, because an in (or out) particle can be in the absolute future of another one due to the tensor product nature of these asymptotic states, bound state equations like the Bethe-Salpeter one have spurious solutions which are excitations in relative energies, the variables conjugate to relative times (which are gauge variables [11])].

IV. ULTRAVIOLET CUTOFF.

As said in Ref. [12,13], the quantization of these rest-frame models has to overcome two problems. On the particle side, the complication is the quantization of the square roots associated with the relativistic kinetic energy terms. On the field side (all physical Hamiltonian are nonlocal and, with the exception of the Abelian case, nonpolynomial), the obstacle is the absence (notwithstanding there is no no-go theorem) of a complete regularization and renormalization procedure of electrodynamics (to start with) in the Coulomb gauge: see Ref. [16] (and its bibliography) for the existing results for QED.

However, as shown in Refs. [11,5] [see their bibliography for the relevant references referring to all the quantities introduced in this Section], the rest-frame instant form of dynamics automatically gives a physical ultraviolet cutoff in the spirit of Dirac and Yukawa: it is the Møller radius [17] $\rho = \sqrt{-W^2}c/P^2 = |\vec{S}|c/\sqrt{P^2}$ ($W^2 = -P^2\vec{S}^2$ is the Pauli-Lubanski Casimir), namely the classical intrinsic radius of the worldtube, around the covariant noncanonical Fokker-Price center of inertia Y^μ , inside which the noncovariance of the canonical center of mass \tilde{x}^μ is concentrated. At the quantum level ρ becomes the Compton wavelength of the isolated system multiplied its spin eigenvalue $\sqrt{s(s+1)}$, $\rho \mapsto \hat{\rho} = \sqrt{s(s+1)}\hbar/M = \sqrt{s(s+1)}\lambda_M$ with $M = \sqrt{P^2}$ the invariant mass and $\lambda_M = \hbar/M$ its Compton wavelength. Therefore, the criticism to classical relativistic physics, based on quantum pair production, concerns the testing of distances where, due to the Lorentz signature of spacetime, one has intrinsic classical covariance problems: it is impossible to localize the canonical center of mass \tilde{x}^μ (also named Pryce center of mass and having the same covariance of the Newton-Wigner position operator) in a frame independent way.

Since ρ describes a nontestable classical short distance region [there is a conceptual connection with the aspect of Mach's principle according to which only relative motions are measurable], it sounds reasonable [13] that for a confined system of effective radius $r_o = 1/\Lambda_{QCD}$ (the fundamental scale of QCD) one has $\rho \leq r_o^2 M = M/\Lambda_{QCD}^2$ [this ensures the mass-spin relation $|\vec{S}| = \alpha'_s M^2 + \alpha_o$ of phenomenological Regge trajectories]. Let us

note that in string theory [18] the relevant dimensional quantity is the tension $T_s = 1/2\pi\alpha'_s$ (the energy per unit length), which, at the quantum level, determines a minimal length $L_s = \sqrt{\hbar/T_s} = \sqrt{2\pi\hbar\alpha'_s} \stackrel{\hbar=1}{=} \sqrt{2\pi\alpha'_s}$ [for a classical string one has $|\vec{S}| \leq \alpha'_s M^2$; a QCD string has $2\pi\alpha'_s \leq r_o^2 = \Lambda_{QCD}^{-2}$].

Let us remember [11] that ρ is also a remnant in flat Minkowski spacetime of the energy conditions of general relativity: since the Møller noncanonical, noncovariant center of energy has its noncovariance localized inside the same worldtube with radius ρ (it was discovered in this way) [17], it turns out that an extended relativistic system with the material radius smaller of its intrinsic radius ρ has: i) the peripheral rotation velocity can exceed the velocity of light; ii) its classical energy density cannot be positive definite everywhere in every frame. Now, the real relevant point is that this ultraviolet cutoff determined by ρ exists also in Einstein's general relativity (which is not power counting renormalizable) in the case of asymptotically flat spacetimes, taking into account the Poincaré Casimirs of its asymptotic ADM Poincaré charges (when supertranslations are eliminated with suitable boundary conditions; let us remark that Einstein and Wheeler use closed universes because they don't want to introduce boundary conditions, but in this way they loose Poincaré charges and the possibility to make contact with particle physics).

By comparison, in string cosmology [18], at the quantum level, the string tension $T_{cs} = 1/2\pi\alpha'_{cs} = L_{cs}^2/\hbar$ gives rise to a minimal length $L_{cs} \stackrel{\hbar=1}{=} \sqrt{2\pi\alpha'_{cs}} \geq L_P$ [$L_P = 1.6 \cdot 10^{-33} cm$ is the Planck length] and is determined by the vacuum expectation value of the background metric of the vacuum (if the ground state is flat Minkowski spacetime), while the grand unified coupling constant α_{GUT} (replacing α_s of QCD) is determined by the vacuum expectation value of the background dilaton field. This minimal length $L_{cs} \geq L_P$ (suppressing the gravitational corrections) could be a lower bound for the Møller radius of an asymptotically flat universe. The upper bound on ρ (namely a physical infrared cutoff) could be the Hubble distance $cH_o^{-1} \approx 10^{28} cm$ considered as an effective radius of the universe. Therefore, it seems reasonable that our physical ultraviolet cutoff ρ is meaningful in the range $L_P \leq L_{cs} < \rho <$

cH_o^{-1} .

Moreover, the extended Heisenberg relations of string theory [18], i.e. $\Delta x = \frac{\hbar}{\Delta p} + \frac{\Delta p}{T_{cs}} = \frac{\hbar}{\Delta p} + \frac{\hbar \Delta p}{L_{cs}^2}$ implying the lower bound $\Delta x > L_{cs} = \sqrt{\hbar/T_{cs}}$ due to the $y + 1/y$ structure, have a counterpart in the quantization of the Møller radius [11]: if we ask that, also at the quantum level, one cannot test the inside of the worldtube, we must ask $\Delta x > \hat{\rho}$ which is the lower bound implied by the modified uncertainty relation $\Delta x = \frac{\hbar}{\Delta p} + \frac{\hbar \Delta p}{\hat{\rho}^2}$. This would imply that the center-of-mass canonical noncovariant (Pryce) 3-coordinate $\vec{z} = \sqrt{P^2}(\vec{x} - \frac{\vec{P}}{P^0} \hat{x}^0)$ [11] cannot become a self-adjoint operator. See Hegerfeldt's theorems (quoted in Refs. [5,11]) and his interpretation pointing at the impossibility of a good localization of relativistic particles (experimentally one determines only a worldtube in spacetime emerging from the interaction region). Since the eigenfunctions of the canonical center-of-mass operator are playing the role of the wave function of the universe, one could also say that the center-of-mass variable has not to be quantized, because it lies on the classical macroscopic side of Copenhagen's interpretation and, moreover, because, in the spirit of Mach's principle that only relative motions can be observed, no one can observe it. On the other hand, if one rejects the canonical noncovariant center of mass in favor of the covariant noncanonical Fokker-Pryce center of inertia Y^μ , $\{Y^\mu, Y^\nu\} \neq 0$, one could invoke the philosophy of quantum groups to quantize Y^μ to get some kind of quantum plane for the center-of-mass description.

V. TETRAD GRAVITY.

The next step of the program is the search of Dirac's observables for classical tetrad gravity in globally hyperbolic asymptotically flat spacetimes $M^4 = \Sigma \times R$ with Σ diffeomorphic to R^3 , so to have the asymptotic Poincaré charges and the same ultraviolet cutoff ρ as for the other interactions.

In Ref. [19] there is a new formulation of tetrad gravity avoiding the use of Schwinger's time gauge condition and, with the technology developed for Yang-Mills theory [5], 13 of its 14 1st class constraints have been Abelianized [the Abelianization of the 6 constraints

generating space-diffeomorphisms and Lorentz rotations has been done in 3-orthogonal coordinates on Σ so that the 3-metric is diagonal]. The last constraint (the superHamiltonian one) becomes an integral equation for the momentum conjugate to the conformal factor of the 3-metric. See Ref. [3] c) for an expanded summary of the results and of the still open problems.

Further problems are how to deparametrize the theory [20], so to reexpress it in the form of parametrized field theories on spacelike hypersurfaces in Minkowski spacetime. This is an extremely important point, because, if we add N scalar particles to tetrad gravity (whose reduction to Dirac's observables should define the N -body problem in general relativity), the deparametrization should be the bridge to the previously quoted theory on spacelike hypersurfaces in Minkowski spacetime [11–13] in the limit of zero curvature. A new formulation of the N -body problem would be relevant to try to understand the energy balance in the emission of gravitational waves from systems like binaries. If it will be possible to find the Dirac observables for the particles, one will understand how to extract from the field theory the covariantization of Newton potential [one expects one scalar and one vector (gravitomagnetism) potential] and a mayor problem will be how to face the expected singularities of the mass-self-energies.

Finally one should couple tetrad gravity to the electromagnetic field, to fermion fields and then to the standard model, trying to make to reduction to Dirac's observables in all these cases.

REFERENCES

- [1] P.A.M.Dirac, Can.J.Math. **2**, 129 (1950); "Lectures on Quantum Mechanics", Belfer Graduate School of Science, Monographs Series (Yeshiva University, New York, N.Y., 1964).
- [2] S.Shanmugadhasan, J.Math.Phys. **14**, 677 (1973). L.Lusanna, Int.J.Mod.Phys. **A8**, 4193 (1993). M.Chaichian, D.Louis Martinez and L.Lusanna, Ann.Phys.(N.Y.)**232**, 40 (1994). L.Lusanna, Phys.Rep. **185**, 1 (1990); Riv. Nuovo Cimento **14**, n.3, 1 (1991); J.Math.Phys. **31**, 2126 (1990); J.Math.Phys. **31**, 428 (1990).
- [3] L.Lusanna, a) "Classical Observables of Gauge Theories from the Multitemporal Approach", talk given at the Conference 'Mathematical Aspects of Classical Field Theory', Seattle 1991, in Contemporary Mathematics **132**, 531 (1992); b) "Hamiltonian Constraints and Dirac's Observables: from Relativistic Particles towards Field Theory and General Relativity", talk at the Workshop "Geometry of Constrained Dynamical Systems", Newton Institute, Cambridge, 1994, (J.M.Charap, Ed.), Cambridge Univ.Press, Cambridge, 1995. c) "Solving Gauss' Laws and Searching Dirac Observables for the Four Interactions", talk at the "Second Conf. on Constrained Dynamics and Quantum Gravity", S.Margherita Ligure 1996 (hep-th/9702114).
- [4] P.A.M.Dirac, Can.J.Phys. **33**, 650 (1955).
- [5] L.Lusanna, Int.J.Mod.Phys. **A10**, 3531 and 3675 (1995).
- [6] L.Lusanna and P.Valtancoli, "Dirac's Observables for the Higgs model: I) the Abelian Case", to appear in Int.J.Mod.Phys. A (hep-th/9606078).
- [7] L.Lusanna and P.Valtancoli, "Dirac's Observables for the Higgs model: II) the non-Abelian SU(2) Case", to appear in Int.J.Mod.Phys. A (hep-th/9606079).
- [8] L.Lusanna and P.Valtancoli, "Dirac's Observables for the SU(3)xSU(2)xU(1) Standard Model", Firenze Univ.preprint, May 1997.

- [9] K.Kuchar, J.Math.Phys. **17**, 777, 792, 801 (1976); **18**, 1589 (1977).
- [10] P.A.M.Dirac, *Rev.Mod.Phys.* **21** (1949) 392.
- [11] L.Lusanna, Int.J.Mod.Phys. **A12**, 645 (1997).
- [12] D.Alba and L.Lusanna, “The Lienard-Wiechert Potential of Charged Scalar Particles and their Relation to Scalar Electrodynamics in the Rest-Frame Instant Form”, Firenze Univ.preprint, May 1997.
- [13] D.Alba and L.Lusanna, “The Classical Relativistic Quark Model in the Rest-Frame Wigner-Covariant Coulomb Gauge”, Firenze Univ.preprint, May 1997.
- [14] F.Bigazzi, R.DePietri and L.Lusanna, in preparation.
- [15] G.Longhi and M.Materassi, “Collective and Relative Variables for a Classical Relativistic Field”, in preparation.
- [16] G.Leibbrandt, “Non-Covariant Gauges”, ch.9 (World Scientific, Singapore, 1994).
- [17] C.Møller, Ann.Inst.H.Poincaré **11**, 251 (1949); “The Theory of Relativity” (Oxford Univ.Press, Oxford, 1957).
- [18] G.Veneziano, “Quantum Strings and the Constants of Nature”, in “The Challenging Questions”, ed.A.Zichichi, the Subnuclear Series n.27 (Plenum Press, New York, 1990).
- [19] L.Lusanna and S.Russo, “Dirac’s Observables for Tetrad Gravity”, in preparation.
- [20] C.J.Isham and K.Kuchar, Ann.Phys.(N.Y.) **164**, 288 and 316 (1984). K.Kuchar, Found.Phys. **16**, 193 (1986).